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**Twin Paradox Lab**

**Physics 205**

**Prof. Singal**

(adopted from M. Trawick)

*The point of this exercise is to step you through a visualization of the twin paradox. We see that a lot of the aging of Bob in Anna’s frame happens when Anna turns around and is not inertial.*

Open the Mathematica file twin\_paradox.nb, hit *Ctrl-A* to select all, and hit *Shift-Enter* to evaluate all the selected statements. Pretend that the time axis has units in years.

1. Here’s the story: Two twins (Bob and Anna) are separated at birth. Bob stays at home, and Anna flies away from the Earth at 0.6c until she reaches a star 30 light years away (as measured by Bob), then turns around and comes back at 0.6c. The red and purple lines in the Minkowski diagram represent the worldlines of Bob and Anna, respectively. From the graph, how long does Anna’s trip *away* from Earth (not counting the return) take according to Bob’s frame? How long according to Anna’s outgoing frame?

2. The blue lines in the graph are “simultaneity lines” in Bob’s frame; everything along a single blue line happens simultaneously according to Bob or any observer in Bob’s frame. According to Bob, when Bob has aged by 25 years, how much has Anna aged by? Answer by following the blue simultaneity line as you switch reference frames with the slider to the frame where Anna is at rest. (An answer by eyeballing the graph is fine.)

3. The green lines in the graph are simultaneity lines in Anna’s outgoing frame. According to Anna, when Anna has aged by 25 years, how much has Bob aged by? Answer by following a green simultaneity line in the frame where Bob is at rest.

4. Are your answers in numbers 2 and 3 the same, or different? Are the roles of Bob and Anna in this series of events symmetric or asymmetric, so far?

5. At this point, turn your attention to the second graph in the file. It shows the same events, but with the origin defined as the event of Anna turning around. By eyeballing the graph, at the instant *just before* Anna turns around, how old is Bob according to Anna’s outgoing frame? (Follow a green simultaneity line in the frame where Bob is at rest from where Anna turns around to Bob, and get the time between the start of the whole thing and then. Note that the start of the whole thing is now at -40 years in Anna’s outgoing frame.)

6. In the second graph, there is an additional cyan line. In which reference frame does this represent a simultaneity line? (Possible choices are Bob’s frame, Anna’s outgoing frame, or Anna’s return frame.)

7. At the instant *just after* Anna turns around, how old is Bob according to Anna? (Follow the cyan simultaneity line in the frame where Bob is at rest from where Anna turns around to Bob, and get the time between the start of the whole thing and then. Note that the start of the whole thing is now at -85 years in Anna’s return frame.)

8. According to Anna, how much has Bob aged during the very short time it took her to turn around?

The non-inertial nature of Anna’s frame means that Bob ages a lot during her turn-around, and this resolves the paradox. When the whole thing is over and Anna returns to Earth, Bob really has aged more than Anna in both frames.